Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2025 Phase-1

[Computer Based Test (CBT) mode]

(Mathematics, Physics and Chemistry)

29/01/2025

Morning

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (MPC) has 25 questions. The maximum marks are 300.

(3) This question paper contains **Three** Parts. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B.**

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt all questions.

(6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section - B (21 – 25) contains** 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

MATHEMATICS

SECTION - A

Multiple Choice uestions:This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by $x\,R\,y$ if and only if $\sec^2 x - \tan^2 y = 1$. Then R is:

(1) both reflexive and transitive but not symmetric

(2) an equivalence relation

(3) reflexive but neither symmetric not transitive

(4) both reflexive and symmetric but not transitive

Answer (2)

Sol. $x\,R\,y : \sec^2 x - \tan^2 y = 1$

Check reflexive:

$x\,R\,x = \sec^2 x - \tan^2 x = 1$

$\forall x \in \left[0, \frac{\pi}{2}\right)$

Check symmetric

$x\,R\,y \Rightarrow y\,R\,x$

$\sec^2 x - \tan^2 y = 1$

$= \sec^2 y - \tan^2 x = (1 + \tan^2 y) - (\sec^2 x - 1)$

$= 2 - (\sec^2 x - \tan^2 y)$

$= 2 - 1 = 1$

$\Rightarrow y\,R\,x$

Check transitive

$x\,R\,y$ and $y\,R\,z$

$\Rightarrow \sec^2 x - \tan^2 y = 1$

$\sec^2 y - \tan^2 z = 1$

Add $\Rightarrow \sec^2 x - \tan^2 z + (\sec^2 y - \tan^2 y) = 2$

$\Rightarrow \sec^2 x - \tan^2 z + 1 = 2 \Rightarrow x\,R\,z$

$\Rightarrow R$ is an equivalence relation.

2. Let ABC be a triangle formed by the lines $7x - 6y + 3 = 0$, $x + 2y - 31 = 0$ and $9x - 2y - 19 = 0$. Let the point (h, k) be the image of the centroid of ΔABC in the line $3x + 6y - 53 = 0$. Then $h^2 + k^2 + hk$ is equal to:

(1) 40

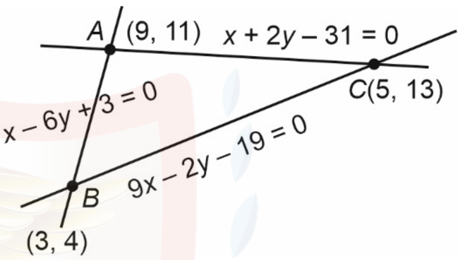
(2) 36

(3) 47

(4) 37

Answer (4)

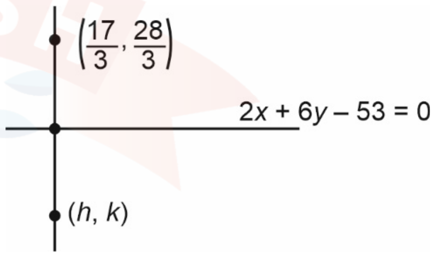
Sol.



Points of intersections are (9, 11), (3, 4), (5, 13)

Centroid of $\Delta ABC = \left(\frac{17}{3}, \frac{28}{3}\right)$

Since during image of ΔABC about line will reflect the whole triangle including centroid, reflected centroid will be image of $\left(\frac{17}{3}, \frac{28}{3}\right)$ about $2x + 6y - 53 = 0$



$$\frac{x - \frac{17}{3}}{2} = \frac{y - \frac{28}{3}}{6} = \frac{-2\left(2\left(\frac{17}{3}\right) + 6\left(\frac{28}{3}\right) - 53\right)}{2^2 + 6^2}$$

$\Rightarrow h = 3, k = 4$

$\Rightarrow h^2 + k^2 + hk = (h + k)^2 - hk$

$= 49 - 12 = 37$

3. Let $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$ and $\left(\vec{a} + \vec{c}\right) \cdot \left(\vec{b} + \vec{c}\right) = 168$. Then the maximum value of $|\vec{c}|^2$ is

(1) 308 (2) 462

(3) 154 (4) 77

Answer (1)

Sol. $\vec{a} \times \vec{c} = \vec{c} \times \vec{b} = -(\vec{b} \times \vec{c})$

$\Rightarrow (\vec{a} \times \vec{c} + \vec{b} \times \vec{c}) = (\vec{a} + \vec{b}) \times \vec{c} = 0$

$\Rightarrow$ $\vec{c}$ is parallel to $\vec{a} + \vec{b}$.

$\vec{c} = \lambda(5\hat{i} + (-6)\hat{j} + 4\hat{k}) \Rightarrow |\vec{c}|^2 = \lambda^2(\pi)$

$(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + |\vec{c}|^2 + \vec{c} \cdot (\vec{a} + \vec{b})$

$= (6 + 5 + 3) + |\vec{c}|^2 + \lambda(25 + 36 + 16) = 168$

$\Rightarrow |\vec{c}|^2 + \lambda(77) = 154$

$\Rightarrow 77\lambda^2 + 77\lambda - 152 = 0$

$\Rightarrow \lambda^2 + \lambda - 2 = 0$

$\Rightarrow (\lambda + 2)(\lambda - 1) = 0$

$|\vec{c}|^2_{max} = (-2)^2(77) = 308$

4. Let $y = y(x)$ be the solution of the differential equation $\cos x\left(\log_e(\cos x)\right)^2 dy$ $+(\sin x - 3y\sin x \log_e(\cos x))dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{4}\right) = \frac{-1}{\log_e 2}$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $-\frac{1}{\log_e(4)}$ (2) $\frac{1}{\log_e(3) - \log_e(4)}$

(3) $\frac{1}{\log_e(4) - \log_e(3)}$ (4) $\frac{2}{\log_e(3) - \log_e(4)}$

Answer (2)

Sol. $\frac{dy}{dx} - \frac{3\sin x}{\cos x \ln\cos x} y = -\frac{\sin x}{\cos x(\ln\cos x)^2}$

I.F. = $e^{-\int \frac{3\tan x}{\ln\cos x}dx}$

Let ln cos$x$ = t

–tan$x$ dx = dt

$e^{3\int \frac{dt}{t}} = e^{3\ln t} = t^3 = (\ln\cos x)^3$

$\therefore$ Solution will be

$y(\ln\cos x)^3 = -\int (\tan x)(\ln\cos x)dx$

$y(\ln\cos x)^3 = \frac{(\ln\cos x)^2}{2} + c$

$\because$ $y\left(\frac{\pi}{4}\right) = -\frac{1}{\ln 2}$

$\Rightarrow c = 0$

$\therefore$ $y = \frac{1}{2(\ln\cos x)}$

$y\left(\frac{\pi}{6}\right) = \frac{1}{2} \times \frac{1}{\ln\left(\cos\frac{\pi}{6}\right)}$

$= \frac{1}{2}\left[\frac{1}{\ln\left(\frac{\sqrt{3}}{2}\right)}\right]$

$= \frac{1}{2} \times \frac{1}{\ln\sqrt{3} - \ln 2}$

$= \frac{1}{\ln 3 - \ln 4}$

5. The value of $\lim_{n\to\infty}\left(\sum_{k=1}^{n} \frac{k^3 + 6k^2 + 11k + 5}{(k+3)!}\right)$ is

(1) $\frac{7}{3}$ (2) $\frac{4}{3}$

(3) $\frac{5}{3}$ (4) 2

Answer (3)

Sol. $\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{k^3+6k^2+11k+5}{(k+3)!}\right)$

$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{k^3+6k^2+11k+6-1}{(k+3)!}\right)$

$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{(k+1)(k+2)(k+3)-1}{(k+3)!}\right)$

$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k!}-\frac{1}{(k+3)!}\right)$

$=\left(\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots\infty\right)-\left(\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\cdots\infty\right)$

$=(e-1)-\left(e-1-\frac{1}{1!}-\frac{1}{2!}-\frac{1}{3!}\right)$

$=1+\frac{1}{2}+\frac{1}{6}=\frac{10}{6}=\frac{5}{3}$

6. Let P be the set of seven digit numbers with sum of their digits equal to 11. If the numbers in P are formed by using the digits 1, 2 and 3 only, then the number of elements in the set P is:

(1) 173 (2) 161
(3) 164 (4) 158

Answer (2)

Sol. Case I: 3 2 2 1 1 1 1

$n_1=\frac{7!}{4!2!}=105$

Case II: 2 2 2 2 1 1 1

$n_2=\frac{7!}{4!3!}=35$

Case III: 3 3 1 1 1 1 1

$n_3=\frac{7!}{2!4!}=21$

Total numbers = 105 + 35 + 21 = 161

7. Let M and m respectively be the maximum and the minimum values of

$$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 4x\\ \sin^2 x & 1+\cos^2 x & 4\sin 4x\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix},\ x\in R$$

Then M^4-m^4 is equal to

(1) 1295 (2) 1040
(3) 1280 (4) 1215

Answer (3)

Sol. $R_1\to R_1-R_2,\ R_2\to R_2-R_3$

$$\begin{vmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ \sin^2 x & \cos^2 x & 1+4\sin 4x\end{vmatrix}=f(x)$$

$C_2\to C_1+C_2$

$$\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ \sin^2 x & 1 & 1+4\sin 4x\end{vmatrix}=f(x)$$

$f(x) = 1 + 4\sin4x+1 = 2 + 4\sin4x$

$M = 6,\ m = -2$

$M^4 - m^4 = 1280$

8. The integral $80\int_0^{\pi/4}\left(\frac{\sin\theta+\cos\theta}{9+16\sin 2\theta}\right)d\theta$ is equal to

(1) $4\log_e 3$ (2) $6\log_e 4$
(3) $2\log_e 3$ (4) $3\log_e 4$

Answer (1)

Sol. $I=\int_0^{\frac{\pi}{4}}\left(\frac{\sin\theta+\cos\theta}{9-16\sin 2\theta}\right)d\theta$

Take $\sin\theta - \cos\theta = t$

$(\cos\theta + \sin\theta)d\theta = dt$

$(\sin\theta - \cos\theta)^2 = t^2$

$\Rightarrow\ \sin2\theta = 1 - t^2$

$\theta = 0 \to t = -1$

$\theta=\frac{\pi}{4}\to t=0$

$I=\int_{-1}^{0}\frac{dt}{9+16(1-t^2)}$

$=\frac{1}{16}\int_{-1}^{0}\frac{dt}{\frac{25}{16}-t^2}$

$=\frac{1}{4}\left[\frac{1}{10}\log\left|\frac{5+4t}{5-4t}\right|\right]_{-1}^{0}$

$=\frac{1}{40}[0+\log_e 9]$

$I=\frac{\log_e 9}{40}$

$80I = 2\log_e 9$

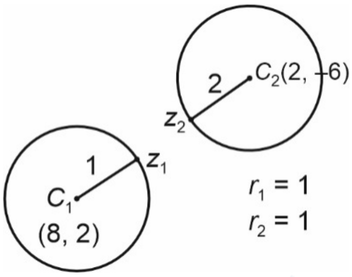
$80I = 4\log_e 3$

9. Let $|z_1 - 8 - 2i| \le 1$ and $|z_2 - 2 + 6i| \le 2$, $z_1, z_2 \in C$. Then the minimum value of $|z_1 - z_2|$ is

(1) 13 (2) 10
(3) 7 (4) 3

Answer (3)

Sol.



$|z_1 - z_2| = C_1C_2 - (r_1 + r_2)$

$= \sqrt{6^2 + 8^2} - (1 + 2)$

$= 10 - 3 = 7$

10. Let $A = [a_{ij}] = \begin{bmatrix} \log_5 128 & \log_4 5 \\ \log_5 8 & \log_4 25 \end{bmatrix}$.

If A_{ij} is the cofactor of $a_{ij}, C_{ij} = \sum_{k=1}^{2} a_{ik} A_{jk}, 1 \le i, j \le 2$

and $C = [C_{ij}]$, then $8|C|$ is equal to

(1) 222 (2) 242
(3) 288 (4) 262

Answer (2)

Sol. $[a_{ij}] = \begin{bmatrix} 7\log_5 2 & \frac{1}{2}\log_2 5 \\ 3\log_5 2 & \log_2 5 \end{bmatrix}$

$[A_{ij}] = \begin{bmatrix} \log_2 5 & -3\log_5 2 \\ -\frac{1}{2}\log_2 5 & 7\log_5 2 \end{bmatrix}$

$C_{ij} = \sum_{k=1}^{2} a_{ik} A_{jk}, \ 1 \le i, j \le 2$

$C = \begin{bmatrix} \frac{11}{2} & 0 \\ 0 & \frac{11}{2} \end{bmatrix}$

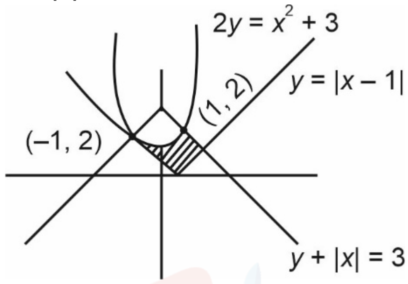
$8|C| = \frac{121}{4} \cdot 8 = 242$

11. Let the area of the region $\{(x, y) : 2y \le x^2 + 3, y + |x| \le 3, y \ge |x - 1|\}$ be A. Then $6A$ is equal to

(1) 18 (2) 12
(3) 16 (4) 14

Answer (4)

Sol.



$$\int_{-1}^{1}\left(\frac{x^2+3}{2} + (x-1)\right)dx + \int_{1}^{2}\left((3-x) - (x-1)\right)dx$$

$$\Rightarrow \left.\frac{x^3}{6} + \frac{3x}{2} + \frac{x^2}{2} - x\right|_{-1}^{1} + \left.3x - \frac{x^2}{2} - \frac{x^2}{2} + x\right|_{1}^{2}$$

$A = \frac{7}{3} \Rightarrow 6A = 14$

12. Let $L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$ and

$L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$ be two lines.

Let L_3 be a line passing through the point (α, β, γ) and be perpendicular to both L_1 and L_2. If L_3 intersects L_1, then $|5\alpha - 11\beta - 8\gamma|$ equals

(1) 18 (2) 25
(3) 16 (4) 20

Answer (2)

Sol. $\frac{x-\alpha}{a} = \frac{y-\beta}{b} = \frac{z-\gamma}{c} : L_3$

$(a\hat{i} + b\hat{j} + c\hat{k}) \,||\, (\hat{i} - \hat{j} + 2\hat{k}) \times (-\hat{i} + 2\hat{j} + \hat{k})$

$(a, b, c) \equiv (5, 3, -1)$

$\Rightarrow \frac{x-\alpha}{5} = \frac{y-\beta}{3} = \frac{z-\gamma}{-1} = \lambda$

Point of intersection

$\alpha = p + 1 - 5\lambda, \beta = -p + 2 - 3\lambda$

$\gamma = 2p + 1 + \lambda$

$\Rightarrow |5\alpha - 11\beta - 8\gamma| = 25$

13. Let $x_1, x_2, ..., x_{10}$ be ten observations such that $\sum_{i=1}^{10}(x_i-2)=30$, $\sum_{i=1}^{10}(x_i-\beta)^2=98, \beta>2$, and their variance is $\frac{4}{5}$. If μ and σ^2 are respectively the mean and the variance of $2(x_1-1)+4\beta, 2(x_2-1)+4\beta, ...,$ $2(x_{10}-1)+4\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to

(1) 110 (2) 90

(3) 100 (4) 120

Answer (3)

Sol. $\sum_{i=1}^{10}(x_i-2)=30$

$\sum_{i=1}^{10}x_i=50$

$\Rightarrow$ Mean = 5

Variance = $\frac{4}{5}=\frac{\sum x_i^2}{10}-(\bar{x})^2$

$\frac{4}{5}=\frac{\sum x_i^2}{10}-25$

$\Rightarrow \sum x_i^2=258$

Now, $\sum_{i=1}^{10}(x_i-\beta)^2=98$

$\sum_{i=1}^{10}x_i^2-2\beta\sum_{i=1}^{10}x_i+10\beta^2=98$

$\Rightarrow 258-2\beta(50)+10\beta^2=98$

$\Rightarrow 10\beta^2-100\beta+160=0$

$\Rightarrow \beta^2-10\beta+16=0$

$\Rightarrow \beta=8$ as $\beta>2$

Now, as per the question

$2(x_1-1)+4\beta, 2(x_2-1)+4\beta, \dots 2(x_{10}-1)+4\beta$

Can be simplified as

$2x_1+30, 2x_2+30, \dots, 2x_{10}+30$

$\mu=2(5)+30=40$

$\sigma^2=2^2\left(\frac{4}{5}\right)=\frac{16}{5}$

$\frac{\beta\mu}{\sigma^2}=\frac{8\times 40}{\frac{16}{5}}=100$

14. The least value of n for which the number of integral terms in the Binomial expansion of $\left(\sqrt[3]{7}+\sqrt[12]{11}\right)^n$ is 183, is

(1) 2184 (2) 2148

(3) 2196 (4) 2172

Answer (1)

Sol. $\left(\sqrt[3]{7}+\sqrt[12]{11}\right)^n$

$T_{K+1}={}^nC_K\,7^{\frac{n-k}{3}}\,11^{\frac{k}{12}}$

12 divides k and 3 divides $n-k$

For integer terms

$\Rightarrow$ Multiples of 12 for k would work

$\Rightarrow k=0, 12, 24$

$\Rightarrow k_{max}=12\times 182=2184$

$\Rightarrow$ Minimum value of n will be 2184

$\Rightarrow$ Option (1) is correct

15. The number of solutions of the equation $\left(\frac{9}{x}-\frac{9}{\sqrt{x}}+2\right)\left(\frac{2}{x}-\frac{7}{\sqrt{x}}+3\right)=0$ is

(1) 4 (2) 3

(3) 1 (4) 2

Answer (1)

Sol. $\left(\frac{9}{x}-\frac{9}{\sqrt{x}}+2\right)\left(\frac{2}{x}-\frac{7}{\sqrt{x}}+3\right)=0$

Let $\frac{1}{\sqrt{x}}=t$

$(9t^2-9t+2)(2t^2-7t+3)=0$

$9t^2 - 9t + 2 = 0$

$\Rightarrow t = \frac{9 \pm \sqrt{81-72}}{18}$

$\Rightarrow t = \frac{9 \pm 3}{18}$

$\Rightarrow t = \frac{2}{3}, \frac{1}{3}$

$\Rightarrow 2t^2 - 7t + 3 = 0$

$\Rightarrow 2t^2 - 6t - t + 3 = 0$

$\Rightarrow 2t(t-3) - 1(t-3) = 0$

$t = \frac{1}{2}, 3$

$\Rightarrow \sqrt{x} = \frac{3}{2}, 3, 2, \frac{1}{3}$

$\Rightarrow x = \frac{9}{4}, 9, 4, \frac{1}{9}$

Hence, number of solutions = 4

16. Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is:

(1) 84 (2) 122

(3) 90 (4) 108

Answer (3)

Sol. Let A.P. be a, $a + d$, $a + 2d$

$3a + 3d = 54$

$a + d = 18$ … (1)

$1600 < \frac{20}{2}[2a + 19d] < 1800$

$160 < 2a + 19d < 180$

$160 < 18 \times 2 + 17d < 180$

$\frac{124}{7} < d < \frac{144}{17}$

$\because d \in$ integer $\Rightarrow d = 8$

$a + d = 18$

$\Rightarrow a = 10$

$a_{11} = a + 10d$

$= 10 + 10 \times 8 = 90$

17. Let the ellipse $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$ and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, $A < B$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be 4. If E_1 and E_2 meet at A, B, C and D, then the area of the quadrilateral $ABCD$ equals

(1) $6\sqrt{6}$ (2) $\frac{24\sqrt{6}}{5}$

(3) $\frac{12\sqrt{6}}{5}$ (4) $\frac{18\sqrt{6}}{5}$

Answer (2)

Sol. $2ae = 4$

$\Rightarrow a = 2\sqrt{3}$

$\Rightarrow 1 - \frac{b^2}{12} = \frac{1}{3} \Rightarrow b^2 = 8$

$\frac{2b^2}{a} \times \frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow \frac{2 \times 8}{2\sqrt{3}} \times \frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow \frac{A^2}{B} = 2 \Rightarrow A^2 = 2B$

$1 - \frac{A^2}{B} = \frac{1}{3}$

$\Rightarrow B = 3 \Rightarrow A^2 = 6$

$E_1 : \frac{x^2}{12} + \frac{y^2}{8} = 1$ …(i)

$E_1 : \frac{x^2}{6} + \frac{y^2}{9} = 1$ …(ii)

On solving (i) & (ii)

$(x, y) = \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right)$

Four points are vertices of rectangle area = $\frac{24\sqrt{6}}{5}$

18. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+7\hat{j}+3\hat{k}$. Let $L_1:\vec{r}=\left(-\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\vec{a},\ \lambda\in\mathbf{R}$ and $L_2:\vec{r}=\left(\hat{j}+\hat{k}\right)+\mu\vec{b},\ \mu\in\mathbf{R}$ be two lines. If the line L_3 passes through the point of intersection of L_1 and L_2 and is parallel to $\vec{a}+\vec{b}$, then L_3 passes through the point

(1) (8, 26, 12) (2) (2, 8, 5)

(3) (5, 17, 4) (4) (–1, –1, 1)

Answer (1)

Sol. $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=2\hat{i}+7\hat{j}+3\hat{k}$

$L_1:\vec{r}=\left(-\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\vec{a},\ \lambda\in R$

and $L_2=\left(\hat{j}+\hat{k}\right)+\mu\vec{b},\mu\in R$

$\vec{a}+\vec{b}=3\hat{i}+9\hat{j}+4\hat{k}$

$L_1:\vec{r}=(\lambda-1)\hat{i}+2(\lambda+1)\hat{j}+(\lambda+1)\hat{k}$

$L_2:\vec{r}=2\mu\hat{i}+(7\mu+1)\hat{j}+(1+3\mu)\hat{k}$

For point of intersection of L_1 & L_2

$\lambda-1=2\mu$ and $2(\lambda+1)=7\mu+1$

$\Rightarrow\ \lambda=3$ and $\mu=1$

$L_3:\vec{r}=2\hat{i}+8\hat{i}+4\hat{k}+\alpha(3\hat{i}+9\hat{j}+4\hat{k})$

For $\alpha=2$

$\vec{r}=8\hat{i}+26\hat{j}+12\hat{k}$

19. Two parabolas have the same focus (4, 3) and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersects at the points A and B, then $(AB)^2$ is equal to :

(1) 392 (2) 192

(3) 384 (4) 96

Answer (2)

Sol. The parabolas are

$(x-4)^2+(y-3)^2=x^2$ …(i)

and $(x-4)^2+(y-3)^2=y^2$ …(ii)

If point of intersection are $A(x_1, y_1)$ and $B(x_2, y_2)$

By solving (i) and (ii), we get

$x_1+x_2=14$ and $x_1x_2=25$

$(AB)^2=2((x_1+x_2)^2-4x_1x_2)=192$

20. Let the line $x+y=1$ meet the circle $x^2+y^2=4$ at the points A and B. If the line perpendicular to AB and passing through the mid-point of the chord AB intersects the circle at C and D, then the area of the quadrilateral $ADBC$ is equal to:

(1) $2\sqrt{14}$

(2) $3\sqrt{7}$

(3) $5\sqrt{7}$

(4) $\sqrt{14}$

Answer (1)

Sol. Solving $x=y$ & $x^2+y^2=4$ gives

$C\left(\sqrt{2},\sqrt{2}\right)$ and $D\left(-\sqrt{2},-\sqrt{2}\right)$

Solving $x+y=1$ & $x^2+y^2=4$ gives

$A\left(\frac{1+\sqrt{7}}{2},\frac{1-\sqrt{7}}{2}\right)$ & $B\left(\frac{1-\sqrt{7}}{2},\frac{1+\sqrt{7}}{2}\right)$

$$\text{Required area}=2\times\frac{1}{2}\begin{vmatrix}\sqrt{2} & \sqrt{2} & 1\\ \frac{1-\sqrt{7}}{2} & \frac{1+\sqrt{7}}{2} & 1\\ -\sqrt{2} & -\sqrt{2} & 1\end{vmatrix}$$

$=2\sqrt{14}$ sq. units

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let [t] be the greatest integer less than or equal to t. Then the least value of $p \in N$ for which

$$\lim_{x\to 0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots\left[\frac{9^2}{x^2}\right]\right)\right)\geq 1$$

is equal to _____.

Answer (24)

Sol. $\lim_{x\to 0^+}\left(x\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots+\left[\frac{9^2}{x^2}\right]\right)\geq 1$

$\Rightarrow$ $(1 + 2 + 3 +\ldots + p) - (1^2 + 2^2 + \ldots + 9^2) \geq 1$

$\Rightarrow$ $\frac{p(p+1)}{2}-\frac{9(10)(19)}{6}\geq 1$

$\Rightarrow$ $p(p + 1) \geq 572$

$\Rightarrow$ Least natural values of p is 24

22. Let $f:(0,\infty)\to \mathbf{R}$ be a twice differentiable function.

If for some $a\neq 0, \int_0^1 f(\lambda x)d\lambda = af(x), f(1)=1$ and $f(16)=\frac{1}{8}$, then $16-f'\left(\frac{1}{16}\right)$ is equal to____.

Answer (112)

Sol. Given, $\int_0^1 f(\lambda x)d\lambda = af(x)$ …(1)

Let $\lambda x = u$

$d\lambda = \frac{1}{x}du$

$\therefore$ From (1) $\frac{1}{x}\int_0^x f(u)du = af(x)$

$\Rightarrow$ $\int_0^x f(u)du = axf(x)$

Differentiate both sides

$f(x) = a(xf'(x) + f(x))$

$\Rightarrow$ $f(x)=axf'(x)+af(x)$

$\Rightarrow$ $(1-a)f(x)=axf'(x)$

$\Rightarrow$ $\frac{f'(x)}{f(x)}=\frac{(1-a)}{a}\cdot\frac{1}{x}$

Integrate both side w.r.t. (x)

$\Rightarrow$ $\int\frac{f'(x)}{f(x)}dx=\frac{(1-a)}{a}\int\frac{1}{x}dx$

$\Rightarrow$ $\ln f(x)=\left(\frac{1-a}{a}\right)\ln x+c$

Now at x = 1 $f(1)$ = 1

$\Rightarrow$ $c = 0$

Also given $f(16)=\frac{1}{8}$

$\therefore$ $\frac{1}{8}=(16)^{\frac{1-a}{a}}$

$\Rightarrow$ $2^{-3}=2^{\frac{4-4a}{a}}$

$\Rightarrow$ $-3=\frac{4-4a}{a}$

$\Rightarrow$ $-3a = 4 - 4a$

$\Rightarrow$ $a = 4$

$\therefore$ $f(x) = x^{-3/4}$

$f(x)=\frac{-3}{4}x^{-\frac{7}{4}}$

Put $x=\frac{1}{16}$

$f'\left(\frac{1}{16}\right)=\frac{-3}{4}\left(\frac{1}{16}\right)^{-7/4}=\frac{-3}{4}\cdot 2^{-4\times\left(\frac{-7}{4}\right)}=-96$

$\therefore$ $16-f'\left(\frac{1}{16}\right)\Rightarrow 16-(-96)=112$

23. Let $S = \{x : \cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x+1)\}$. Then $\sum_{x\in S}(2x-1)^2$ is equal to_____.

Answer (5)

Sol. $\cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x+1)$.

$$\frac{\pi}{2} - \sin^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x+1)$$

$$-\frac{\pi}{2} - 2\sin^{-1} x = \sin^{-1}(2x+1)$$

$$\sin\left(-\frac{\pi}{2} - 2\sin^{-1} x\right) = \sin(\sin^{-1}(2x+1))$$

$-\cos(2\sin^{-1}x) = (2x + 1)$

$-(1 - 2x^2) = 2x + 1$

$-1 + 2x^2 = 2x + 1$

$2x^2 - 2x - 2 = 0$

$$x = \frac{2 \pm \sqrt{4+16}}{4}$$

$$x = \frac{2 \pm 2\sqrt{5}}{4} = \frac{1 \pm \sqrt{5}}{2}\left\{x = \frac{1+\sqrt{5}}{2} \text{ rejected}\right\}$$

So, $\sum_{x\in s}(2x-1)^2 = 5$

24. The number of 6 letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice is

Answer (1405)

Sol. $^5C_3 \times \frac{6!}{2!2!2!} + {}^5C_2\left(\frac{6!}{2!4!} \times 2 + \frac{6!}{3!3!}\right) + {}^5C_1 \cdot 1$

$= 10 \times 90 + 10(15 \times 2 + 20) + 5$

$= 900 + 500 + 5$

$= 1405$

25. Let $S = \left\{m \in \mathbf{Z} : A^{m^2} + A^m = 3I - A^{-6}\right\}$, where $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$. Then $n(S)$ is equal to______.

Answer (2)

Sol. $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

Now finding characteristic equation

$$\begin{vmatrix} 2-\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $(2 - \lambda)(-\lambda) - (-1)(1) = -2\lambda + \lambda^2 + 1 = 0$

$\Rightarrow$ $\lambda^2 - 2\lambda + 1 = 0$

$\Rightarrow$ $(\lambda - 1)^2 = 0$

$\Rightarrow$ $\lambda = 1$

Since A satisfies $(A - I)^2 = 0$

$\therefore$ $A = I + N$ where

$N = A - I$

$$N = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$N^2 = 0$

$A^m = (I + N)^m = I + mN$

$A^m \cdot A^m = (I + mN)(I + mN) = I + 2mN + m^2N^2$

Since $N^2 = 0$

$\Rightarrow$ $A^{m^2} = I + 2mN$

Now putting in given condition

$I + m^2N + I + mN = 3I - A^{-6}$ ...(i)

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

$A^{-6} = (A^{-1})^6 = I + (-6)N$

$\therefore$ Putting in (i)

$(m^2 + m)N = I - (I - 6N)$

$(m^2 + m)N = 6N$

Since $N \neq 0$

$\Rightarrow$ $m^2 + m = 6$

$\Rightarrow$ $m^2 + m - 6 = 0$

$\Rightarrow$ $(m - 2)(m + 3) = 0$

$\Rightarrow$ $m = 2, -3$

$\therefore$ Number of elements in S is 2

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. The workdone in an adiabatic change in an ideal gas depends upon only :

(1) Change in its volume

(2) Change in its temperature

(3) Change in its pressure

(4) Change in its specific heat

Answer (2)

Sol. Work done in adiabatic process = $\frac{nR\Delta T}{1-\gamma}$

So, depends upon change in temperature.

27. Two projectiles are fired with same initial speed from same point on ground at angles of (45° – α) and (45° + α), respectively, with the horizontal direction. The ratio of their maximum heights attained is :

(1) $\frac{1+\sin 2\alpha}{1-\sin 2\alpha}$ (2) $\frac{1+\sin\alpha}{1-\sin\alpha}$

(3) $\frac{1-\tan\alpha}{1+\tan\alpha}$ (4) $\frac{1-\sin 2\alpha}{1+\sin 2\alpha}$

Answer (4)

Sol. $H_{max} = \frac{u^2\sin^2\theta}{2g}$

$$\frac{H_1}{H_2} = \frac{\sin^2(45-\alpha)}{\sin^2(45+\alpha)}$$

$$= \frac{(\cos\alpha-\sin\alpha)^2}{(\cos\alpha+\sin\alpha)^2}$$

$$\frac{H_1}{H_2} = \frac{1-\sin 2\alpha}{1+\sin 2\alpha}$$

28. Consider a long straight wire of a circular cross-section (radius *a*) carrying a steady current *I*. The current is uniformly distributed across this cross-section. The distances from the centre of the wire's cross-section at which the magnetic field [inside the wire, outside the wire] is half of the maximum possible magnetic field, anywhere due to the wire, will be

(1) [a/2, 2a] (2) [a/2, 3a]

(3) [a/4, 2a] (4) [a/4, 3a/2]

Answer (1)

Sol. Magnetic field inside cylinder at distance *r* from axis

$$= \frac{\mu_0 I}{\pi R^2}\,\frac{r}{2}$$

$$B_{max} = \frac{\mu_0 I}{2\pi R}$$

$$\frac{\mu_0 I}{2\pi R^2} r_1 = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi R}\right)$$

$$r_1 = \frac{R}{2} = \frac{a}{2}$$

Magnetic field outside cylinder at distance *r* from axis

$$= \frac{\mu_0 I}{2\pi r}$$

$$\frac{\mu_0 I}{2\pi r_2} = \frac{\mu_0 I}{\pi R}\left(\frac{1}{2}\right)$$

$r_2 = 2R = 2a$

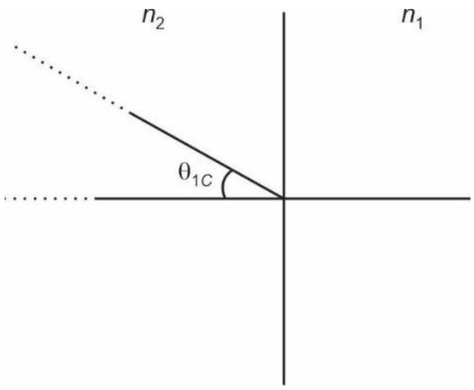
29. At the interface between two materials having refractive indices n_1 and n_2, the critical angle for reflection of an em wave is θ_{1C}. The n_2 material is replaced by another material having refractive index n_3 such that the critical angle at the interface between n_1 and n_3 materials is θ_{2C}. If $n_3 > n_2 > n_1$; $\frac{n_2}{n_3} = \frac{2}{5}$ and $\sin\theta_{2C} - \sin\theta_{1C} = \frac{1}{2}$, then θ_{1C} is

(1) $\sin^{-1}\left(\frac{1}{3n_1}\right)$ (2) $\sin^{-1}\left(\frac{1}{6n_1}\right)$

(3) $\sin^{-1}\left(\frac{5}{6n_1}\right)$ (4) $\sin^{-1}\left(\frac{2}{3n_1}\right)$

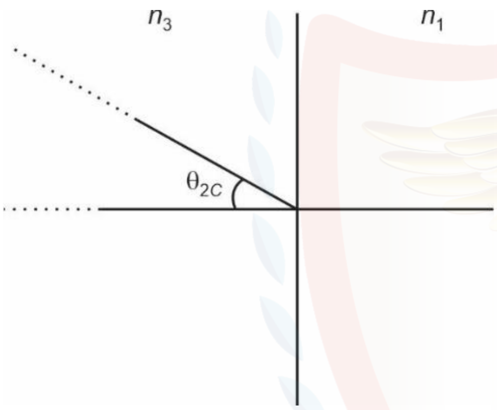
Answer (*)

Sol.



$n_2 \sin(\theta_{1C}) = 1$

$\sin(\theta_{1C}) = \frac{n_1}{n_2}$



$n_3 \sin(\theta_{2C}) = 1$

$\Rightarrow \sin(\theta_{2C}) = \frac{n_1}{n_3}$

Also, $n_3 = \frac{5n_2}{2} \Rightarrow \sin(\theta_C) = \frac{2n_1}{5n_2}$

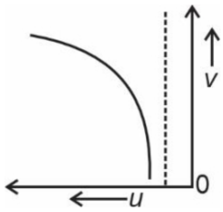
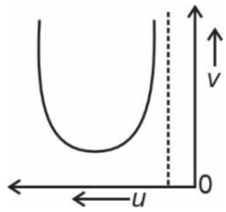
$\Rightarrow \frac{n_1}{n_2} = \frac{5}{2}.\sin(\theta_{2C}) \Rightarrow \sin(\theta_{2C}) - \sin(\theta_{1c}) = \frac{1}{2}$

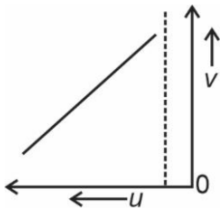
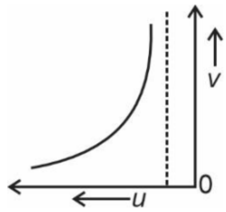
Given, $\frac{2n_1}{5n_2} - \frac{n_1}{n_2} = \frac{1}{2} \Rightarrow \frac{n_1}{n_2}\left(\frac{-3}{5}\right) = \frac{1}{2}$

Coming out to be (–ve)

*** None of the answer is matching**

30. Let u and v be the distances of the object and the image from a lens of focal length f. The correct graphical representation of u and v for a convex lens when $|u| > f$, is

(1)  (2) 

(3)  (4) 

Answer (4)

Sol. Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$

Since $|\mu| > f$

So, RHS is positive.

31. The pair of physical quantities not having same dimensions is :

(1) Angular momentum and Planck's constant

(2) Torque and energy

(3) Surface tension and impulse

(4) Pressure and Young's modulus

Answer (3)

Sol. [Angular momentum] = ML^2T^{-1}

[Planck's Constant] = ML^2T^{-1}

[Torque] = ML^2T^{-2}

[Energy] = ML^2T^{-2}

[Surface tension] = MT^{-2}

[Impulse]= MLT^{-1}

[Pressure] = $ML^{-1}T^{-2}$

[Young's modulus] = $ML^{-1}T^{-2}$

32. The expression given below shows the variation of velocity (v) with lime (t), $v = At^2 + \frac{Bt}{C+t}$. The dimension of ABC is :

(1) $[M^0L^1T^{-2}]$ (2) $[M^0L^1T^{-3}]$

(3) $[M^0L^2T^{-3}]$ (4) $[M^0L^2T^{-2}]$

Answer (3)

Sol. $v = At^2 + \frac{Bt}{C+t}$

$[v] = [A]\, t^2 = \left[\frac{Bt}{C+t}\right]$

$[A] = LT^{-3}$

$[C] = T$

$[B] = LT^{-1}$

$[ABC] = L^2T^{-3}$

33. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Choke coil is simply a coil having a large inductance but a small resistance. Choke coils are used with fluorescent mercury-tube fittings. If household electric power is directly connected to a mercury tube, the tube will be damaged.

Reason (R) : By using the choke coil, the voltage across the tube is reduced by a factor $\left(R/\sqrt{R^2+\omega^2L^2}\right)$, where ω is frequency of the supply across resistor R and inductor L. If the choke coil were not used, the voltage across the resistor would be the same as the applied voltage.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

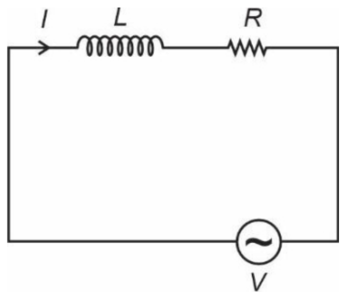
(2) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of (A)

(3) **(A)** is false but **(R)** is true

(4) **(A)** is true but **(R)** is false

Answer (1)

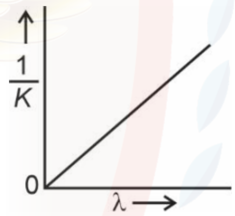
Sol.

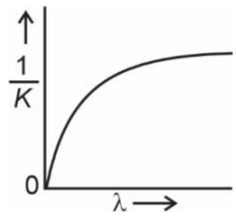


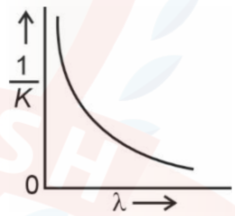
$I = \frac{V}{\sqrt{R^2+L^2\omega^2}}$

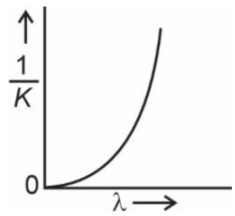
$V_R = \frac{VR}{\sqrt{R^2+L^2\omega^2}}$

34. If λ and K are de Broglie wavelength and kinetic energy, respectively, of a particle with constant mass. The correct graphical representation for the particle will be

(1)


(2)


(3)


(4)


Answer (4)

Sol. $\lambda = \frac{h}{\sqrt{2mK}}$

$\lambda^2 = \frac{h^2}{2mK}$

$\frac{1}{K} = \left(\frac{2m}{h^2}\right)\lambda^2$

35. Consider I_1 and I_2 are the currents flowing simultaneously in two nearby coils 1 & 2, respectively. If L_1 = sell inductance of coil 1, M_{12} = mutual inductance of coil 1 with respect to coil 2, then the value of induced emf in coil 1 will be

(1) $\varepsilon_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$

(2) $\varepsilon_1 = -L_1\frac{dI_2}{dt} - M_{12}\frac{dI_1}{dt}$

(3) $\varepsilon_1 = -L_1\frac{dI_1}{dt} + M_{12}\frac{dI_2}{dt}$

(4) $\varepsilon_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_1}{dt}$

Answer (1)

Sol. Magnitude of induced emf due to self inductance

$= \frac{LdI_1}{dt}$

Magnitude of induced emf due to mutual inductance

$= \frac{MdI_2}{dt}$

36. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Electromagnetic waves carry energy but not momentum.

Reason (R) : Mass of a photon is zero.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Answer (2)

Sol. EM wave carry both energy and momentum. Rest mass of photon is zero.

37 An electric dipole of mass m, charge q, and length l is placed in a uniform electric field $E = E_0\hat{i}$. When the dipole is rotated slightly from its equilibrium position and released, the time period of its oscillations will be :

(1) $2\pi\sqrt{\frac{ml}{2qE_0}}$

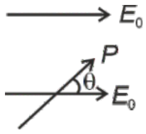
(2) $\frac{1}{2\pi}\sqrt{\frac{ml}{2qE_0}}$

(3) $2\pi\sqrt{\frac{ml}{qE_0}}$

(4) $\frac{1}{2\pi}\sqrt{\frac{2ml}{qE_0}}$

Answer (1)

Sol.



$\tau = PE_0\sin\theta$

If θ is small

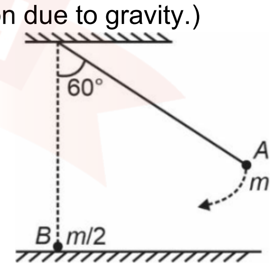
$\tau = -(PE_0)\theta$

$I = m\left(\frac{l}{2}\right)^2\cdot 2 = \frac{ml^2}{2}$

$T = 2\pi\sqrt{\frac{ml^2}{2\cdot PE_0}} = 2\pi\sqrt{\frac{ml^2}{2\cdot qlE_0}}$

$T = 2\pi\sqrt{\frac{ml}{2qE_0}}$

38. As shown below, bob A of a pendulum having massless string of length 'R' is released from 60° to the vertical. It hits another bob B of half the mass that is at rest on a frictionless table in the center. Assuming elastic collision, the magnitude of the velocity of bob A after the collision will be (take g as acceleration due to gravity.)



(1) $\frac{1}{3}\sqrt{Rg}$

(2) $\sqrt{Rg}$

(3) $\frac{4}{3}\sqrt{Rg}$

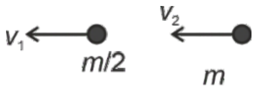
(4) $\frac{2}{3}\sqrt{Rg}$

Answer (1)

Sol. Velocity of A before collision $= \sqrt{2gh}$

$$= \sqrt{2g \times \frac{R}{2}} = \sqrt{Rg}$$

After collision



COM

$$mu = \frac{m}{2}v_1 + mv_2$$

$2u = v_1 + 2v_2$ …(i)

$e = 1$, $u = v_1 - v_2$ …(ii)

$u = 3v_2$

$$v_2 = \frac{u}{3} = \frac{1}{3}\sqrt{Rg}$$

39. Match **List-I** with **List-II**.

	List-I		List-II
(A)	Electric field inside (distance $r > 0$ from center) of a uniformly charged spherical shell with surface charge density σ, and radius R.	(I)	σ/ε_0
(B)	Electric field at distance $r > 0$ from a uniformly charged infinite plane sheet with surface charge density σ.	(II)	$\sigma/2\varepsilon_0$
(C)	Electric field outside (distance $r > 0$ from center) of a uniformly charged spherical shell with surface charge density σ, and radius R.	(III)	0
(D)	Electric field between 2 oppositely charged infinite plane parallel sheets with uniform surface charge density σ.	(IV)	$\frac{\sigma}{\varepsilon_0 r^2}$

Choose the **correct** answer from the options given below:

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(2) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

Answer (*)

Sol. Inside uniformly charged spherical shell, $E = 0$

$\therefore$ A $\rightarrow$ III

For uniformly charged infinite plate

$$E = \frac{\sigma}{2\varepsilon_0}$$

B $\rightarrow$ II

Outside of spherical shell

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{R^2\sigma}{\varepsilon_0 r^2}$$

None of the option is matching for C.

Between two plates $E = \frac{\sigma}{\varepsilon_0}$

D $\rightarrow$ I

None of the option is correct

40. The fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the depth of 2.5 km below the sea level is ______ %. Given, the Bulk modulus of water = 2×10^9 Nm^{-2}, density of water = 10^3 kg m^{-3}, acceleration due to gravity = g = 10 ms^{-2}.

(1) 1.25

(2) 1.5

(3) 1.0

(4) 1.75

Answer (1)

Sol. $B = \frac{\Delta P}{-\left(\frac{\Delta V}{V}\right)}$

$$-\left(\frac{\Delta V}{V}\right) = \frac{\Delta P}{B} = \frac{\rho g h}{B}$$

$$= \frac{10^3 \times 10 \times 2.5 \times 10^3}{2 \times 10^9} = 1.25\%$$

41. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Emission of electrons in photoelectric effect can be suppressed by applying a sufficiently negative electron potential to the photoemissive substance.

Reason (R) : A negative electric potential, which stops the emission of electrons from the surface of a photoemissive substance, varies linearly with frequency of incident radiation.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **(A)** is true but **(R)** is false
(2) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**
(3) **(A)** is false but **(R)** is true
(4) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

Answer (4)

Sol. Negative potential will slow the electrons and if it is sufficient, it will make the photocurrent zero.

$eVs = hf - \phi_0$

42. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Time period of a simple pendulum is longer at the top of a mountain than that at the base of the mountain.

Reason (R) : Time period of a simple pendulum decreases with increasing value of acceleration due to gravity and vice-versa.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **(A)** is false but **(R)** is true
(2) **(A)** is true but **(R)** is false
(3) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**
(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Answer (4)

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

At top of mountain $g\downarrow$, $\therefore\ T\uparrow$

43. A coil of area A and N turns is rotating with angular velocity ω in a uniform magnetic field B about an axis perpendicular to B. Magnetic flux φ and induced emf ε across it, at an instant when B is parallel to the plane of coil, are :

(1) $\varphi = 0, \varepsilon = 0$
(2) $\varphi = AB, \varepsilon = 0$
(3) $\varphi = AB, \varepsilon = NAB\omega$
(4) $\varphi = 0, \varepsilon = NAB\omega$

Answer (3)

Sol. $\phi = NBA\cos\theta$

$\varepsilon = -\frac{d\phi}{dt} = -NBA\frac{d\cos\theta}{dt}$

$\theta = \omega t$

$\varepsilon = -NBA\omega\sin\omega t$

if B is parallel to plane of coil

$\theta = 90°$

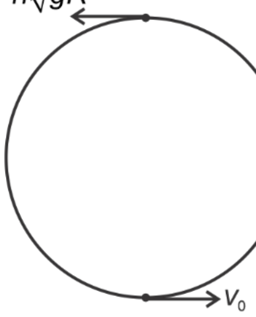
$\phi = 0, E = BA\omega N$

44. A body of mass 'm' connected to a massless and unstretchable string goes in verticle circle of radius 'R' under gravity g. The other end of the string is fixed at the center of circle. If velocity at top of circular path is $n\sqrt{gR}$, where, $n \geq 1$, then ratio of kinetic energy of the body at bottom to that at top of the circle is

(1) $\frac{n^2+4}{n^2}$
(2) $\frac{n}{n+4}$
(3) $\frac{n+4}{n}$
(4) $\frac{n^2}{n^2+4}$

Answer (1)

Sol. $v = n\sqrt{gR}$

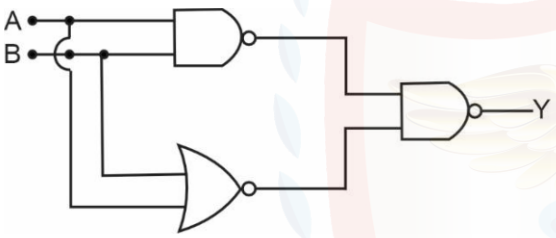


$$v_0 = \sqrt{v^2 + 2g(2R)}$$

$$v_0 = \sqrt{n^2gR + 4gR}$$

$$\therefore \quad \frac{k_{bottom}}{k_{top}} = \frac{v_0^2}{v^2} = \frac{n^2+4}{n^2}$$

45.



For the circuit shown above, equivalent GATE is :

(1) AND gate

(2) OR gate

(3) NOT gate

(4) NAND gate

Answer (2)

Sol. $Y = \overline{\overline{AB}\left(\overline{A+B}\right)}$

$= AB + A + B$

$= A(B+1) + B$

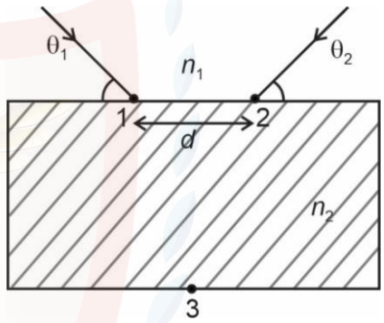
$= A + B$

or GATE

SECTION - B

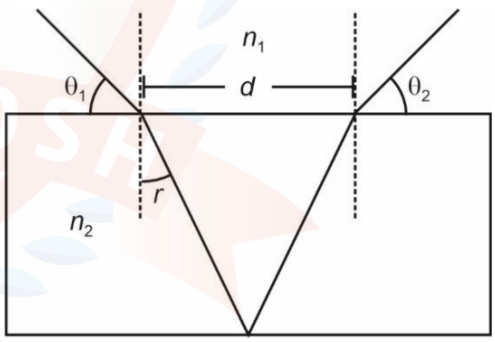
Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. Two light beams fall on a transparent material block at point 1 and 2 with angle θ_1 and θ_2, respectively, as shown in figure. After refraction, the beams intersect at point 3 which is exactly on the interface at other end of the block. Given : the distance between 1 and 2, $d = 4\sqrt{3}$ cm and $\theta_1 = \theta_2 = \cos^{-1}\left(\frac{n_2}{2n_1}\right)$, where refractive index of the block n_2 > refractive index of the outside medium n_1, then the thickness of the block is ________ cm.



Answer (6)

Sol.



$$n_1\sin(90-\theta_1) = n_2\sin r$$

$$n_1 \times \frac{n_2}{2n_1} = n_2 \sin r$$

$$\sin r = \frac{1}{2}$$

$r = 30°$

$$\tan r = \left(\frac{d/2}{t}\right)$$

$$t = \frac{d}{2\tan r} = \frac{d\sqrt{3}}{2} = \frac{(4\sqrt{3})\sqrt{3}}{2}$$

= 6 cm

47. A container of fixed volume contains a gas at 27° C. To double the pressure of the gas, the temperature of gas should be raised to ____ °C.

Answer (327)

Sol. V = constant

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$P_2 = 2P_1$

$T_2 = 2T_1$

= 2 × 300 = 600 K

∴ = 327°C

48. The maximum speed of a boat in still water is 27 km/h. Now this boat is moving downstream in a river flowing at 9 km/h. A man in the boat throws a ball vertically upwards with speed of 10 m/ s. Range of the ball as observed by an observer at rest on the river bank, is _________ cm. (Take g = 10 m/s^2)

Answer (2000)

Sol. $v_y = 10$ m/s

$v_x = v_{river} + v_{boat} = 27 + 9$

= 36 km/h = 10 m/s

$$R = v_x\left(\frac{2v_y}{g}\right) = \frac{2\times 10 \times 10}{10} = 20 \text{ m}$$

= 2000 cm

49. In a hydraulic lift, the surface area of the input piston is 6 cm^2 and that of the output piston is 1500 cm^2. If 100 N force is applied to the input piston to raise the output piston by 20 cm, then the work done is _________ kJ.

Answer (5)

Sol. According to Pascal's law

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1\left(\frac{A_2}{A_1}\right) = \frac{1500}{6}\times 100\text{N}$$

$$W = F_2 d = \frac{1500}{6}\times 100 \times \frac{20}{100}$$

= 5 kJ

50. The coordinates of a particle with respect to origin in a given reference frame is (1, 1, 1) meters. If a force of $F = \hat{i} - \hat{j} + \hat{k}$ acts on the particle, then the magnitude of torque (with respect to origin) in z-direction is _________.

Answer (2)

Sol. $\tau = r \times F$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

Along $\hat{k}$

$\tau_2 = -2\hat{k}$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. For a $Mg|Mg^{2+}(aq) || Ag^{+}(aq) | Ag$ the correct Nernst equation is :

(1) $E_{cell} = E^{\circ}_{cell} + \frac{RT}{2F} \ln \frac{[Ag^{+}]^2}{[Mg^{2+}]}$

(2) $E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F} \ln \frac{[Ag^{+}]^2}{[Mg^{2+}]}$

(3) $E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F} \ln \frac{[Ag^{+}]}{[Mg^{2+}]}$

(4) $E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F} \ln \frac{[Mg^{2+}]}{[Ag^{+}]}$

Answer (1)

Sol.

Cathode	$(Ag^{+}(aq) + e^{-} \rightarrow Ag) \times 2$	
Anode	Mg	$\rightarrow Mg^{2+}(aq) + 2e^{-}$
Cell reaction	$2Ag^{+} + Mg$	$\rightarrow 2Ag + Mg^{2+}$

$$Q = \frac{[Mg^{2+}]}{[Ag^{+}]^2}$$

By Nernst equation

$$E_{cell} = E^{o}_{cell} - \frac{RT}{nF} \ln Q$$

$$E_{cell} = E^{o}_{cell} - \frac{RT}{nF} \ln \frac{[Mg^{2+}]}{[Ag^{+}]^2}$$

$$= E^{o}_{cell} + \frac{RT}{2F} \ln \frac{[Ag^{+}]^2}{[Mg^{2+}]}$$

52. 1.24 g of AX_2 (molar mass 124 g mol^{-1}) is dissolved in 1 kg of water to form a solution with boiling point of 100.0156°C, while 25.4 g of AY_2 (molar mass 250 g mol^{-1}) in 2 kg of water constitutes a solution with a boiling point of 100.0260°C.

$K_b(H_2O) = 0.52$ K kg mol^{-1}

Which of the following is **correct** ?

(1) AX_2 and AY_2 (both) are fully ionised

(2) AX_2 is fully ionised while AY_2 is completely unionised

(3) AX_2 and AY_2 (both) are completely unionised

(4) AX_2 is completely unionised while AY_2 is fully ionised

Answer (2)

Sol. For AX_2

$\Delta T_b = iK_b m$

$0.0156 = i \times 0.52 \times \frac{1.24}{124 \times 1}$

$3 = i$

$3 = 1 + 2\alpha$

$1 = \alpha$

For AY_2

$\Delta T_b = iK_b m$

$0.0260 = i \times 0.52 \times \frac{25.4}{250 \times 2}$

$i \approx 1$

$\therefore$ AX_2 is completely ionised & AY_2 is completely unionised

53. The standard reduction potential values of some of the p-block ions are given below. Predict the one with the strongest oxidising capacity.

(1) $E_{Al^{3+}/Al} = -1.66$ V

(2) $E_{Sn^{4+}/Sn^{2+}} = +1.15$ V

(3) $E_{Tl^{3+}/Tl} = +1.26$ V

(4) $E_{Pb^{4+}/Pb^{2+}} = +1.67$ V

Answer (4)

Sol. The element having strongest oxidising capacity will have highest value of standard reduction potential

54. The reaction $A_2 + B_2 \rightarrow 2AB$ follows the mechanism

$$A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} A + A \text{ (fast)}$$

$$A + B_2 \xrightarrow{k_2} AB + B \text{ (slow)}$$

$A + B \rightarrow AB$ (fast)

The overall order of the reaction is :

(1) 1.5 (2) 2.5

(3) 3 (4) 2

Answer (1)

Sol. Since, second step is slow step

$r = k_2[A][B_2]$ …(i)

Also, $\frac{k_1}{k_{-1}} = \frac{[A]^2}{[A_2]}$

$[A] = \left(\frac{[A_2]k_1}{k_{-1}}\right)^{\frac{1}{2}}$ …(ii)

$r = k_2\left(\frac{k_1}{k_{-1}}\right)^{\frac{1}{2}}[A_2]^{\frac{1}{2}}[B_2]$

order $= \frac{1}{2} + 1 = \frac{3}{2}$

55. The molar conductivity of a weak electrolyte when plotted against the square root of its concentration, which of the following is expected to be observed?

(1) A small increase in molar conductivity is observed at infinite dilution

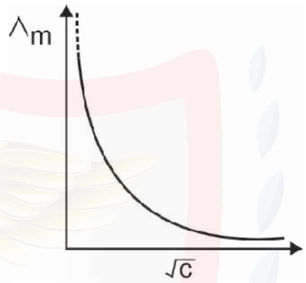
(2) Molar conductivity increases sharply with increase in concentration

(3) A small decrease in molar conductivity is observed at infinite dilution

(4) Molar conductivity decreases sharply with increase in concentration.

Answer (4)

Sol. For weak electrolyte, variation of $\wedge_m$ with $\sqrt{c}$ is



56. At temperature T, compound $AB_2(g)$ dissociates as $AB_2(g) \rightleftharpoons AB(g) + \frac{1}{2}B_2(g)$ having degree of dissociation x (small compared to unity). The correct expression for x in terms of K_p and p is

(1) $\sqrt[3]{\frac{2K_p^2}{p}}$ (2) $\sqrt[3]{\frac{2K_p}{p}}$

(3) $\sqrt[4]{\frac{2K_p}{p}}$ (4) $\sqrt{K_p}$

Answer (1)

Sol. $AB_2(g) \rightleftharpoons AB(g) + \frac{1}{2}B_2(g)$

$t = 0$	p_0		
$t = t_{eq}$	$p_0(1-x)$	p_0x	$\frac{p_0x}{2}$

$$p = p_0x - p_0x + p_0x + \frac{p_0x}{2}$$

$$p = p_0\left(1 + \frac{x}{2}\right)$$

$$p_0 = \frac{p}{\left(1+\frac{x}{2}\right)}$$

$$K_p = \frac{(p_{AB})(p_{B_2})^{1/2}}{(p_{AB_2})}$$

$$K_p = \frac{(p_0 x)\left(\frac{p_0 x}{2}\right)^{1/2}}{p_0(1-x)}$$

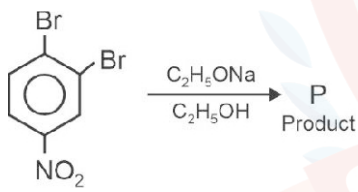
$$K_p = \frac{\frac{px}{\left(1+\frac{x}{2}\right)}\left(\frac{p}{1+\frac{x}{2}} \times \frac{x}{2}\right)^{1/2}}{\frac{p(1-x)}{\left(1+\frac{x}{2}\right)}}$$

Since x << 1

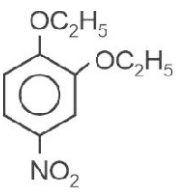
$$K_p = \frac{p^{1/2} x^{3/2}}{2^{1/2}}$$

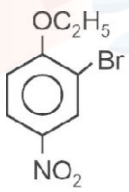
$$x = \sqrt[3]{\frac{2K_p^2}{p}}$$

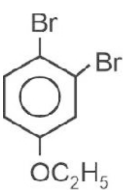
57. In the following substitution reaction :

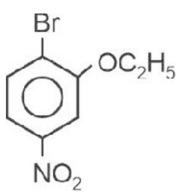


Product 'P' formed is

(1) 

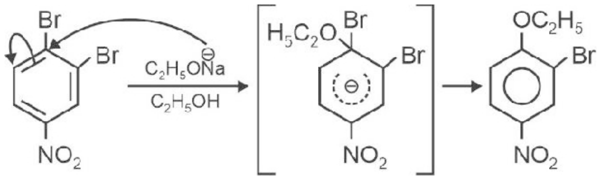
(2) 

(3) 

(4) 

Answer (2)

Sol. Br at the para of NO_2 will undergo aromatic nucleophilic substitution by nucleophile C_2H_5ONa.



58. Total number of nucleophiles from the following is

NH_3, PhSH, $(H_3C)_2S$, $H_2C{=}CH_2$, $\bar{O}H$, H_3O^+, $(CH_3)_2CO$, $\rangle{=}NCH_3$

(1) 4
(2) 7
(3) 5
(4) 6

Answer (3)

Sol. Any specie having electrons available for donation can act as nucleophile.

Total 5 are present.

59. Choose the **correct** statements :

(A) Weight of a substance is the amount of matter present in it.

(B) Mass is the force exerted by gravity on an object.

(C) Volume is the amount of space occupied by a substance.

(D) Temperatures below 0°C are possible in Celsius scale, but in Kelvin scale negative temperature is not possible.

(E) Precision refers to the closeness of various measurements for the same quantity.

Choose the **correct** answer from the options given below.

(1) (A), (B) and (C) only
(2) (A), (D) and (E) only
(3) (C), (D) and (E) only
(4) (B), (C) and (D) only

Answer (3)

Sol. Mass of substance is amount of matter present in it. Weight is force exerted by gravity on object.

60. Match **List-I** with **List-II**.

List-I (Carbohydrate)		**List-II (Linkage Source)**
(A) Amylose	(I)	β-C_1-C_4, plant
(B) Cellulose	(II)	α-C_1-C_4, animal
(C) Glycogen	(III)	α-C_1-C_4, α-C_1-C_6, plant
(D) Amylopectin	(IV)	α-C_1-C_4, plant

Choose the **correct** answer from the options given below.

(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(2) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(3) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Answer (4)

Sol. Amylose ⇒ It is a plant based starch it has α-C_1-C_4 glycosidic linkage.

Cellulose ⇒ It has β-C_1-C_4 glycosidic linkage

Glycogen ⇒ It has α-C_1-C_4 and glycosidic linkage (animal starch)

Amylopectin ⇒ It is a plant based with α-C_1-C_4 and C_1-C_6 glycosidic linkage

61. If a_0 is denoted as the Bohr radius of hydrogen atom, then what is the de-Broglie wavelength (λ) of the electron present in the second orbit of hydrogen atom? [n : any integer]

(1) $\frac{2a_0}{n\pi}$ (2) $\frac{4n}{\pi a_0}$

(3) $\frac{4\pi a_0}{n}$ (4) $\frac{8\pi a_0}{n}$

Answer (4)

Sol. $r_n = \frac{a_0 n^2}{z}$

Also, $2\pi r_n = n\lambda$

Where λ is de-Broglie wavelength

$$\frac{2\pi a_0 n^2}{z} = n\lambda$$

For second orbit of H-atom

$$\lambda = \frac{8\pi a_0}{n}$$

62. Match **List - I** with **List - II.**

List - I (Complex)		**List - II (Hybridisation & Magnetic characters)**
(A) $[MnBr_4]^{2-}$	(I)	d^2sp^3 & diamagnetic
(B) $[FeF_6]^{3-}$	(II)	sp^3d^2 & paramagnetic
(C) $[Co(C_2O_4)_3]^{3-}$	(III)	sp^3 & diamagnetic
(D) $[Ni(CO)_4]$	(IV)	sp^3 & paramagnetic

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(2) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Answer (1)

Sol. $[MnBr_4]^{2-}$

Mn^{2+}, Br^- is WFL

d^5, so it is sp^3 and paramagnetic.

$[FeF_6]^{3-}$

Fe^{3+}, F^- is WFL

d^5, so it is sp^3d^2 and paramagnetic.

$[Co(C_2O_4)_3]^{3-}$

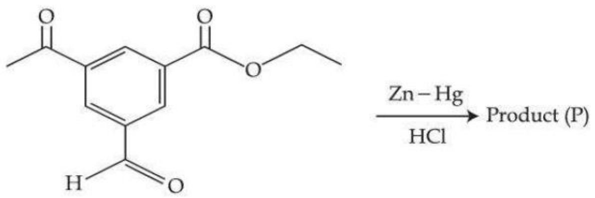
Co^{3+}, $C_2O_4^{2-}$ is a SFL

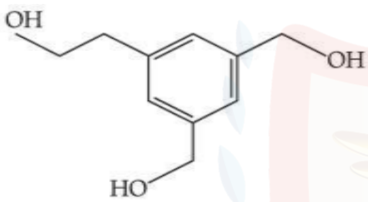
d^6, so it is d^2sp^3 and diamagnetic.

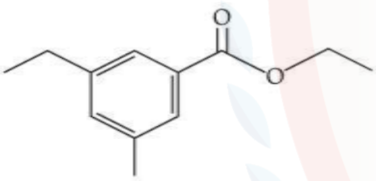
$[Ni(CO)_4]$, CO is a SFL

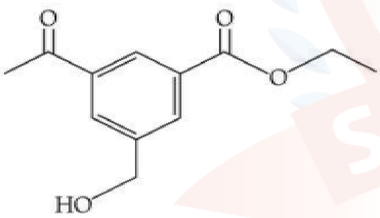
d^{10}, it is sp^3 and diamagnetic.

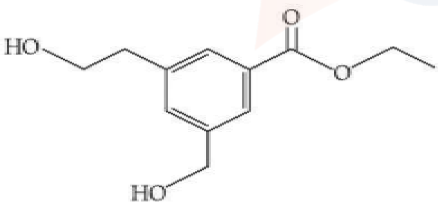
63. The product (P) formed in the following reaction is:



(1) 

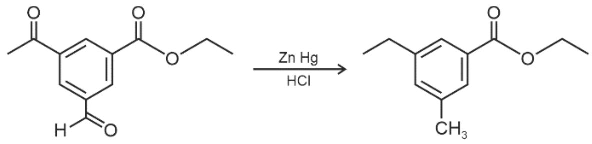
(2) 

(3) 

(4) 

Answer (2)

Sol. It is Clemmensen reduction, it will not reduce ester,



Ester cannot be reduced by clemmensen reduction

64. The correct increasing order of stability of the complexes based on Δ_0 value is:

I. $[Mn(CN)_6]^{3-}$ II. $[Co(CN)_6]^{4-}$

III. $[Fe(CN)_6]^{4-}$ IV. $[Fe(CN)_6]^{3-}$

(1) III < II < IV < I (2) II < III < I < IV

(3) IV < III < II < I (4) I < II < IV < III

Answer (4)

Sol. Neglecting pairing energy

I. $[Mn(CN)_6]^{3-} \Rightarrow Mn^{3+}$, t_{2g}^4, CFSE = $-0.4 \times 4\ \Delta_o = -1.6\Delta_o$

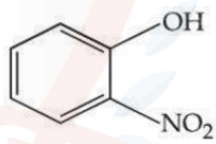
II. $[Co(CN)_6]^{4-} \Rightarrow Co^{2+}$, $t_{2g}^6\, e_g^1$, CFSE = $-0.4 \times 6 + 0.6 \times 1 = -1.8\Delta_o$

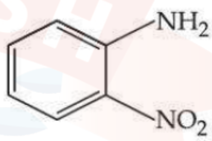
III. $[Fe(CN)_6]^{4-} \Rightarrow Fe^{2+}$, $t_{2g}^6\, e_g^0$, CFSE = $-0.4 \times 6 = -2.4\Delta_o$

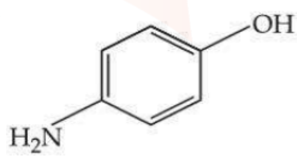
IV. $[Fe(CN)_6]^{3-} \Rightarrow Fe^{3+}$, $t_{2g}^5\, e_g^0$, CFSE = $-0.4 \times 5 = -2\Delta_o$

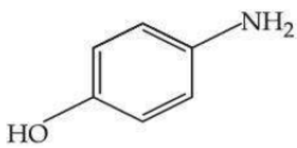
Order of stability III > IV > II > I

65. The steam volatile compounds among the following are :

(A) 

(B) 

(C) 

(D) 

Choose the **correct** answer from the options given below :

(1) (B) and (D) Only

(2) (A) and (C) Only

(3) (A), (B) and (C) Only

(4) (A) and (B) Only

Answer (4)

Sol. Ortho nitro phenol and ortho nitro aniline will be steam volatile as they will show intra molecular H-bonding.

66. Match List I with List-II

	List-I (Structure)		**List-II (IUPAC Name)**
(A)	$H_3C-CH_2-CH(C_2H_5)-CH_2-CH(CH_3)-C_2H_5$	(I)	4-Methylpent-l-ene
(B)	$(CH_3)_2C(C_3H_7)_2$	(II)	3-Ethyl-5-methylheptane
(C)		(III)	4, 4-Dimethylheptane
(D)		(IV)	2-Methyl-1, 3-pentadiene

Choose the correct answer from the options given below

(1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(2) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

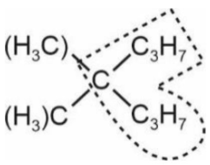
(3) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Answer (2)

Sol. (A) $CH_3-CH_2-CH(C_2H_5)-CH_2-CH(CH_3)-C_2H_5$

3-Ethyl-5-methylheptane

(B) 

4,4-Dimethylheptane

(C) 

2-Methyl-1,3-pentadiene

(D) 

4-Methylpent-1-ene

67. 500 J of energy is transferred as heat to 0.5 mol of Argon gas at 298 K and 1.00 atm. The final temperature and the change in internal energy respectively are: Given: R = 8.3 JK^{-1} mol^{-1}

(1) 378 K and 500 J (2) 368 K and 500 J

(3) 378 K and 300 J (4) 348 K and 300 J

Answer (4)

Sol. At cons. P

$Q = nC_p\,\Delta T$

$500 = 0.5 \times \frac{5}{2} \times 8.3\left(T_f - 298\right)$

$346.2\text{ K} = T_f$

$\Delta U = nC_V\Delta T$

$= \frac{1}{2} \times \frac{3}{2} \times 8.3 \times \left(346.2 - 298\right)$

= 300 J

68. The correct option with order of melting points of the pairs (Mn, Fe), (Tc, Ru) and (Re, Os) is:

(1) Fe < Mn, Ru < Tc and Re < Os

(2) Mn < Fe, Tc < Ru and Os < Re

(3) Fe < Mn, Ru < Tc and Os < Re

(4) Mn < Fe, Tc < Ru and Re < Os

Answer (2)

Sol. Melting point order

Fe > Mn

Ru > Tc

Re > Os

69. An element 'E' has the ionisation enthalpy value of 374 kJ mol^{-1}.'E' reacts with elements A, B, C and D with electron gain enthalpy values of –328, –349, –325 and –295 kJ mol^{-1}, respectively. The correct order of the products EA, EB, EC and ED in terms of ionic character is:

(1) ED > EC > EA > EB (2) EA > EB > EC > ED

(3) EB > EA > EC > ED (4) ED > EC > EB > EA

Answer (3)

Sol. The element having high value of Electron gain enthalpy (magnitude) will form a compound having higher ionic character so order of ionic character

EB > EA > EC > ED

70. Given below are two statements :

Statement (I) : The radii of isoelectronic species increases in the order.

$Mg^{2+} < Na^{+} < F^{-} < O^{2-}$

Statement (II) : The magnitude of electron gain enthalpy of halogen decreases in the order.

Cl > F > Br > I

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) **Statement I** is correct but **Statement II** is incorrect

(2) Both **Statement I** and **Statement II** are correct

(3) **Statement I** is incorrect but **Statement II** is **correct**

(4) Both **Statement I** and **Statement II** are incorrect

Answer (2)

Sol. $r \propto q^{-}$ (for isoelectronic species)

$\propto \frac{1}{q^{+}}$

$\therefore$ Statement I is correct

Magnitude of electron gain enthalpy

Cl > F > Br > I

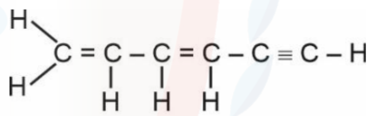
SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. The sum of sigma (σ) and pi(π) bonds in Hex-1, 3-dien-5-yne is ______.

Answer (15)

Sol.



Hex-1, 3-dien-5-yne

No. of π bond = 4

No. of σ bond = 11

72. If A_2B is 30% ionised in an aqueous solution, then the value of van't Hoff factor (i) is ______ × 10^{-1}.

Answer (16)

Sol.

AB_2		A^{2+}	$\overset{-}{+}$	$2B$
1		0		0
$1-\alpha$	2	α		α

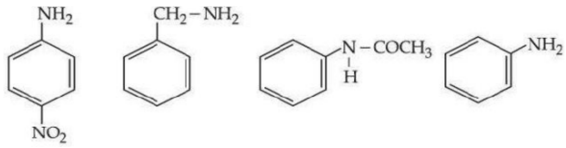
$i = 1 + 2\alpha$

$= 1 + 2 \times (0.3)$

$= 1.6$

$= 16 \times 10^{-1}$

73. Given below are some nitrogen containing compounds

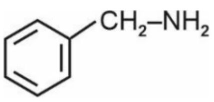


Each of them is treated with HCl separately, 1.0 g of the most basic compound will consume _____ mg of HCl.

(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16, Cl : 35.5)

Answer (341)

Sol. The most basic compound will be aliphatic amine due to localised electrons



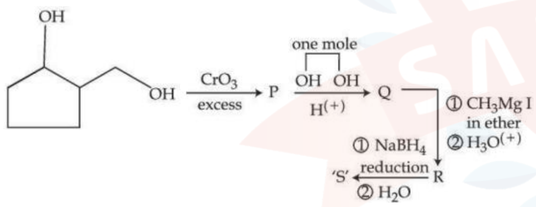
1 mole of this will consume 1 mole HCl

So mass of HCl consumed for 1 g of this compound

$= \frac{1}{107} \times 36.5$

= 0.341 gm

= 341 mg

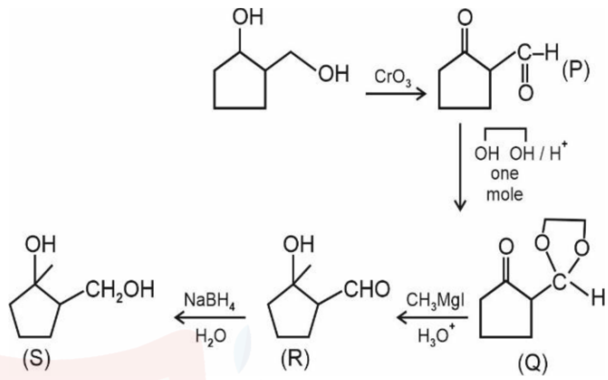
74. 

0.1 mole of compound 'S' will weight ______ g.

(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16)

Answer (13)

Sol.



mass of 0.1 mole (S) = 0.1(84 + 32 +14)

= 13 g

75. The molar mass of the water insoluble product formed from the fusion of chromite ore ($FeCr_2O_4$), with Na_2CO_3 in presence of O_2 is _____ g mol^{-1}.

Answer (160)

Sol. $FeCr_2O_4 + Na_2CO_3 + O_2 \rightarrow Na_2CrO_4 + Fe_2O_3 + CO_2$

Insoluble product will be Fe_2O_3

molar mass = 56 × 2 + 16 × 3

= 112 + 48

= 160

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